Investigation into the Impacts of Self-Inductance and Mutual Inductance on Saggy Winding Inductive Coupling Power Transmission

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Abstract

This paper investigates the effects of self-inductance and mutual inductance on saggy coil magnetic resonant inductive coupling power transmission (IPT). The simulation of the saggy coil is conducted using a specialist tool called IPTVisual to visualise the phenomenon of saggy winding in the system. The results provide important intuitions for optimising saggy coil inductive coupling devices for better performance and reliability in a variety of applications. The setup of simulations and experimental shows a slight difference. The mutual inductance and coupling coefficient of two saggy coils are analysed and discussed. There is a difference of 1.74% in self-inductance in saggy coil with two fixed mounting point of 1 m distance when comparison is made between simulation and experimental results.

1 Introduction

In power transmission systems, an inductive coupling system is one of the most commonly used in terms of near-field power transfer systems. In a resonant system, the self-inductance, mutual inductance along coupling coefficient contribute significantly to the power transfer efficiency, stability and transmitting power performance.

Inductive coupling concepts revolve around the principle of electromagnetic. It uses induced electromagnetic force to transfer energy between transmitter and receiver coils. The individual self-inductance of each coil plays a major role in the influences of one coil's magnetic field on another.

Introducing the saggy coils in the system changes the outcomes of the inductive coupling significantly. Saggy coils contribute to the irregularities in the magnetic field distribution. This affects the self-inductance as well as the mutual inductance between transmitter and receiver coils. Subsequently, this leads to the challenges such as the potential for energy losses.

The hanging cable derivation is derived from analysing it as a physical problem. The only forces acting on a hanging cable at a certain point are its weight and the tension in the cable. It must be considered that the resultant of these forces equals zero when the cable is at rest. By the sum of these forces being known, a differential equation is created, resulting in the unique solution of cosine hyperbolic [1].

2 Catenary Equations

The condition when the coil in an inductive coupling power transfer (IPT) is loosened or saggy can significantly affect the self-inductance and mutual inductance. The catenary effect shows this phenomenon of line sagging in power transmission lines. This could be applied to the concept of inductive coupling power transmission where the power transfer efficiency is dependent on the configuration of the transmitter and receiver coils. When the coil is tightly wound, the magnetic field lines generated by the current flowing are concentrated within the coil which subsequently offers a higher self-inductance. However, the magnetic field lines become less concentrated when these coils are not tightly wound in which case the coil lines are sagging. As a result, the self-inductance would be reduced significantly depending on the condition of the saggy coils. The self-inductance of a straight cylindrical wire has been discussed in [2].

$$L_{wire}(l,r) = 2\left(l \ln \frac{m+r}{r} - m + \frac{l}{4} + r\right) \tag{1}$$

l is the length of the wire, r is the cross-sectional radius of the wire, and $m = \sqrt{l^2 + r^2}$.

Based on the equation of self-inductance in Eq. 1, the mutual inductance then can be evaluated. In inductive coupling, mutual inductance plays an important aspect in determining the effective power transfer efficiency between transmitter and receiver coils. In saggy coil configuration, the coil becomes loosely wound resulting in the change in proximity and alignment of the coils hence affecting the mutual inductance. The mutual inductance between any two arbitrary windings in free space, C_1 and C_2, is given by Neumann's Formula.

$$M = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l_1} d\vec{l_2}}{D} \tag{2}$$

μ_0 is vacuum permeability. $\vec{l_1}$ and $\vec{l_2}$ are the two differential elements being integrated from the two windings, respectively. $D(\vec{l_1}, \vec{l_2})$ is the distance between $\vec{l_1}$ and $\vec{l_2}$.

$$y = a\cosh\left(\frac{x}{a}\right) \tag{3}$$

Cosh is the hyperbolic cosine function. The scaling factor a represents the ratio between the horizontal tension component exerted on the coil and the weight of the winding per unit length when two arbitrary mounting points are determined. When a is predetermined, the shape of the catenary is established. In this paper, a is treated as a constant on the spacing between these two mounting points. In this study, the scenario where the catenary is tangent to the x-axis (the ground) is focused, thus necessitating the subtraction of a to derive the equation will be employed in the analysis.

$$z = a\cosh\left(\frac{r-r_0}{a}\right) + z_0 \tag{4}$$

Case 1: Knowing two mounting points $\langle x_1,y_1,z_1\rangle$ and $\langle x_2,y_2,z_2\rangle$, and constant a, solving constants r_0 and z_0.

$$\cosh^2(x) - \sinh^2(x) = 1 \tag{5}$$

Arc length:

$$\begin{aligned} l &= \int \sqrt{1+\sinh^2\left(\frac{r-r_0}{a}\right)}\, dr \\ &= \int \cosh\left(\frac{r-r_0}{a}\right)\, dr \\ &= a\sinh\left(\frac{r-r_0}{a}\right) \end{aligned} \tag{6}$$

Case 2: Knowing two mounting points $\langle x_1,y_1,z_1\ \rangle$ and $\langle x_2,y_2,z_2\rangle$, and line length l, solving constants a, then converted to Case 1.

Solution vector:

$$v = \begin{pmatrix} r_0 \\ a \end{pmatrix} \tag{7}$$

$$\begin{cases} f^1(v) = a\cosh\left(\frac{\Delta r-r_0}{a}\right) - a\cosh\left(\frac{-r_0}{a}\right) - \Delta z \\ f^2(v) = a\sinh\left(\frac{\Delta r-r_0}{a}\right) - a\sinh\left(\frac{-r_0}{a}\right) - l \end{cases} \tag{8}$$

$$f = (f^1, f^2) : \mathbb{R}^2 \to \mathbb{R}^2 \tag{9}$$

f^1 comes from the catenary line equation with two points (r_1, z_1) and (r_2, z_2), f^2 comes from the arc length of the catenary line in Equation 8.

The numerical solution is based on Newton's Method for two variables. Initial guess to help solution converge:

$$v_0 = \left(\frac{\Delta r}{2} \mid 0.5\right) \tag{10}$$

Newton function of $f : N = N_f : \mathbb{R}^2 \to \mathbb{R}^2$

$$N(v) = v - J_v^{-1} f(v) \tag{11}$$

where J_v is the Jacobian matrix of f at v.

$$J_v = \left.\begin{pmatrix} f^1_{r_0} & f^1_a \\ f^2_{r_0} & f^2_a \end{pmatrix}\right|_v \tag{12}$$

where

$$f^1_{r_0} = -\sinh\left(\frac{\Delta r - r_0}{a}\right) - \sinh\left(\frac{r_0}{a}\right) \tag{13}$$

$$f^2_{r_0} = -\cosh\left(\frac{\Delta r - r_0}{a}\right) + \cosh\left(\frac{r_0}{a}\right) \tag{14}$$

$$f^1_a = \frac{A - B + C}{a} \tag{15}$$

$$f^2_a = \frac{D - E - F}{a} \tag{16}$$

Table 1 List of equations used in (15) and (16).

A:	$a(\cosh\frac{\Delta r-r_0}{a} - \cosh\left(\frac{r_0}{a}\right))$
B:	$(\Delta r - r_0)\sinh\left(\frac{r-r_0}{a}\right)$
C:	$(\Delta r - r_0)\sinh\left(\frac{r-r_0}{a}\right)$
D:	$a(\sinh(\frac{\Delta r-r_0}{a}) + \sinh(\frac{r_0}{a}))$
E:	$(\Delta r - r_0)\cosh(\frac{\Delta r-r_0}{a})$
F:	$r_0\cosh\left(\frac{r_0}{a}\right)$

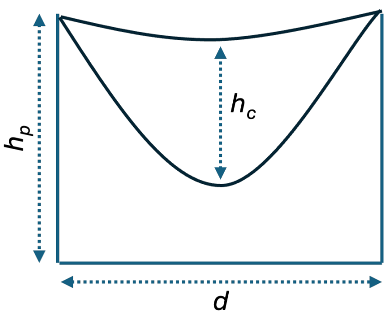

Fig. 1 Illustration of saggy winding with two fixed points at z-axis (d is the distance between two poles, h_p is the total height of poles, and h_c is the gap between top and bottom part of the winding).

3 Simulations

Implementing the catenary equation and the outcome of the simulation is conducted using a specialist tool based on this paper [3]. Two arbitrary mounting points along the z-axis are determined for the simulation of saggy coil configurations. The self-inductance is then evaluated based on the parameter of interest which is the wire cross-sectional area of 0.385 mm with 5 coil turns. The coupling coefficient between the transmitter and receiver coils is then observed when these two mounting points are increased. Subsequently, the positions of mounting points would influence the self-inductance of the saggy coils. Fig. 2 illustrates the variation of saggy coils corresponding to different distances between the two arbitrary points along the z-axis.

In Tab. 2, l represents the coil length, a_{top} denotes the saggy line at the top, a_{bottom} signifies the saggy line at the bottom, and d_z indicates the distance between the two mounting points. Number of coil turns, $N = 5$. Unit for l, z, d_z, a_{top} and a_{bottom} are in m.

Table 2 Simulation results (l, z, a_{top}, a_{bot} are in m, L in μH).

Coil Shape	l	z	d_z	a_{top}	a_{bot}	L
Catenary Saggy Coil with fixed z (Fig. 2)	15	(1,1)	1	1	2	57.78
			0.75	0.75	2.25	57.54
			0.5	0.5	2.5	54.55
Catenary Saggy Coil with varied z (Fig. 3a)	15	(1, 0.2)	1	1.3	1.7	47.41
		(1, 0.3)	1	1.3	1.7	43.53
		(1, 0.4)	1	1.3	1.7	41.55
		(1, 0.5)	1	1.3	1.7	40.30
		(1, 0.6)	1	1.3	1.7	39.46
		(1, 0.7)	1	1.3	1.7	38.90
		(1, 0.8)	1	1.3	1.7	38.54
		(1, 0.9)	1	1.3	1.7	38.33
		(1, 1)	1	1.3	1.7	38.26
Square Coil (Fig. 3b)	18.5	(1, 1)	1	1.1	1	67.51
Saggy Square Coil (Fig. 3c)	21	(1,1)	1	1.1	1.1	84.68

4 Experimental Setup

The experiment setup consists of a coil purposely set up for saggy conditions to meet the catenary concept. Fig. 4 shows the litz wire of 5 number of coil turns. Each turn is set to be 1 cm gap between each and held in place using Kapton tape. The experimental setup is depicted in Fig. 5 where two mounting points are positioned as the anchor points holding up the coil to form the saggy condition.

The distance between the mounting ground and the two mounting points is kept constant at 1 m. Initially, the distance between the two mounting points d_z is set to 1 m which then gradually decreased to 750 mm, 700 mm, 500 mm and 250 mm to observe the changes in self-inductance. Combinations of z positions are set to observe the variations of mounting points as in Fig. 3a for example in the simulation setup.

The first set is labelled as transmitter whilst the second set of saggy coil is prepared as receiver and self-inductance is measured individually. These two saggy coils are then connected

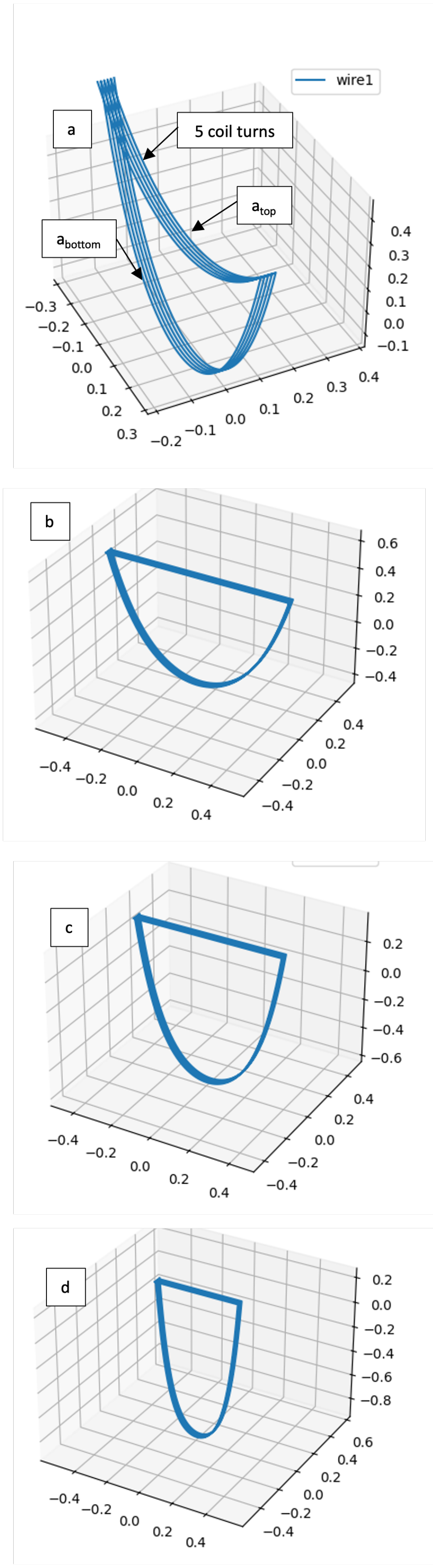

Fig. 2 a) $d = 1$ m, $a_{top} = 1.2$ m, $a_{bot} = 2$ m, b) $d = 1$ m, $a_{top} = 1$ m, $a_{bot} = 2$ m, c) $d = 0.75$ m, $a_{top} = 0.75$ m, $a_{bot} = 2.25$ m, d) $d = 0.5$ m, $a_{top} = 0.5$ m, $a_{bot} = 2.5$ m.

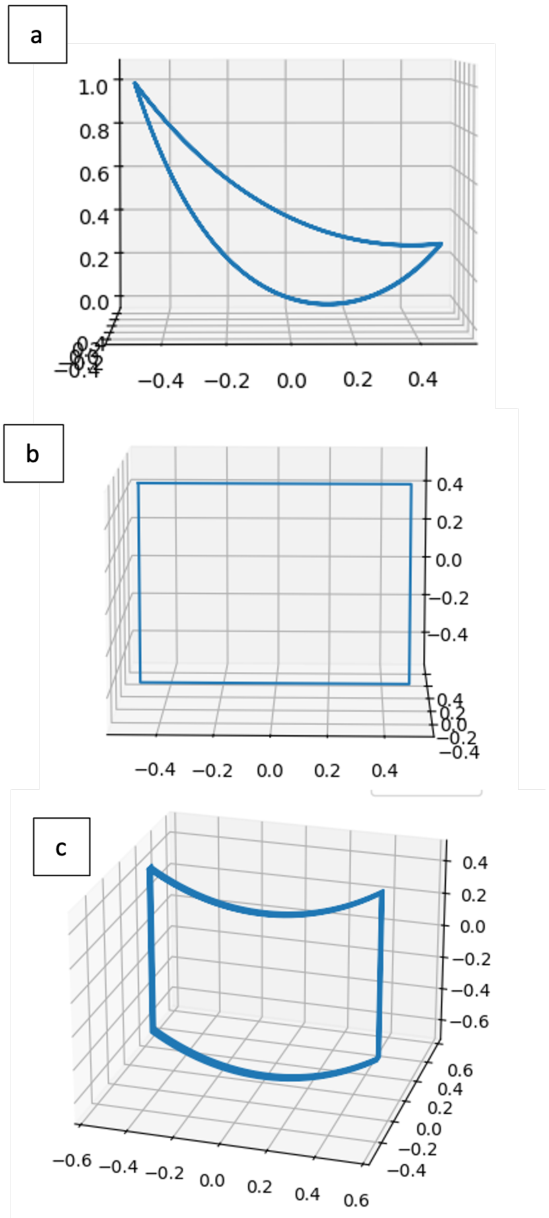

Fig. 3 Catenary saggy coil with; a) Varied two z points, b) Square coil, c) Saggy square coil.

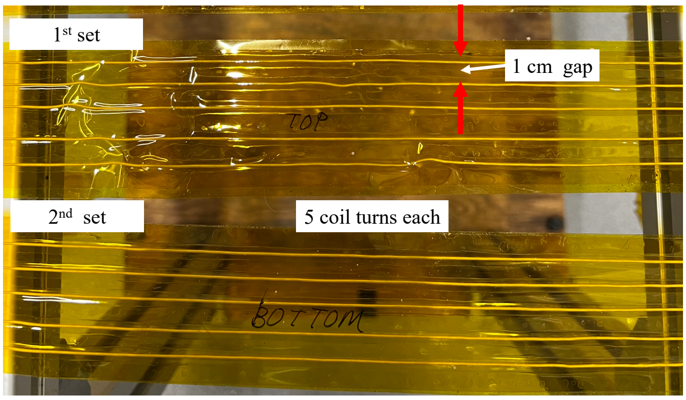

Fig. 4 Litz wire with wire cross-sectional radius of 0.385 mm, and total coil length of 15 m. The wire with N = 5 is kept together using Kapton tape to prevent entanglement while keeping the gap between turns of 10 mm.

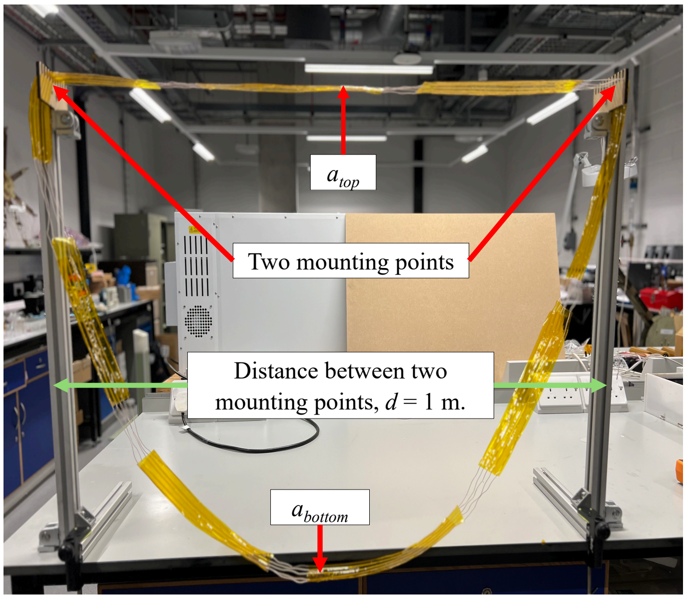

Fig. 5 Experimental setup to measure self-inductance of saggy winding of two mounting points at z-axis with varying distance between two poles. Distance between two mounting points, d = 1000 mm. d is decreased gradually to observe the changes in self-inductance.

to the drive board to induce the voltage and form the resonant inductive coupling. The system is operated in resonance frequency, f_s = 515 kHz. The mutual inductance between two saggy coils is then measured at the open circuit. Fig. 6 depicts an abstract of an experimental setup example for saggy coils in IPT system (L_1 and L_2 are the self-inductance for primary and secondary coils, respectively whilst M_{12} is the mutual inductance).

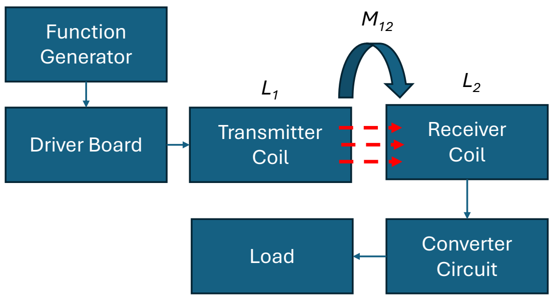

Fig. 6 Block diagram of experimental setup example for the saggy coils.

5 Results

The outcomes of simulation setups have been validated with the experimental results. Fig. 7 shows a bar chart comparison of self-inductance in simulation and measured in experimental

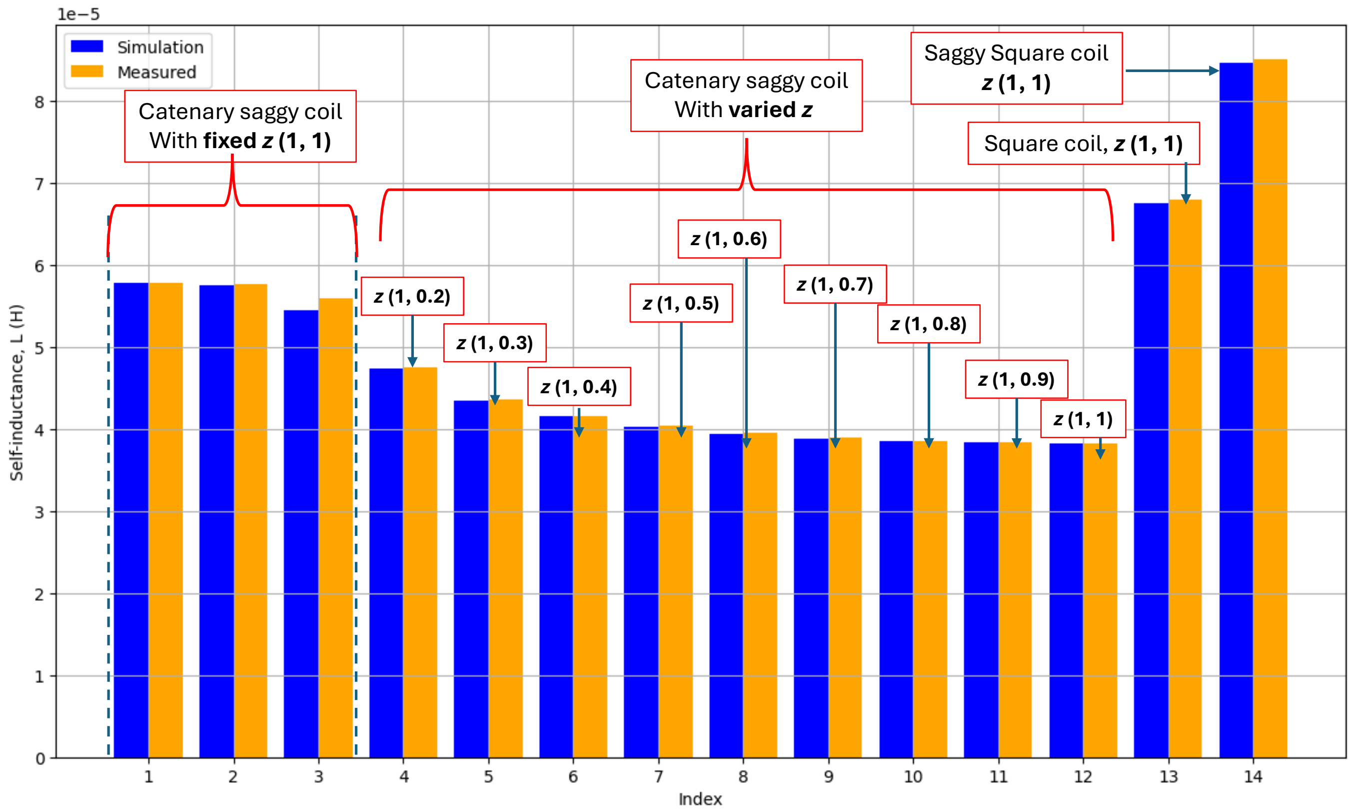

Fig. 7 A bar chart showing the comparison of self-inductance in simulation and measured in experimental setup for the saggy coils and the square saggy coils.

setup for the saggy coils and the square saggy coils. The results are divided into four main groups which are the catenary saggy coil with fixed z, the catenary saggy coil with varied sz, and the ideal square coil without saggy and saggy square coil.

There is a difference of 1.74% in self-inductance when taken into a saggy coil with two fixed mounting points of 1 m distance when a comparison is made between simulation and experimental results (57.78 μH and 58.78 μH respectively). Tab. 2 shows the self-inductance decreased when d_z is reduced gradually. When d_z is decreased from 1 m to 750 mm, L is 57.54 μH and dropped to 57.55 μH when d_z is positioned at 500 mm of distance. The mutual inductance M are 1.612 μH, 1.606 μH, 1.606 μH respectively.

In catenary saggy coils with varied z, only one mounting point is fixed whilst the other is reduced gradually. At (z_1, z_2) = (1, 1), L is measured at 38.26 μH. When z_2 is at 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9, the self-inductance indicated as 47.41 μH, 43.53 μH, 41.55 μH, 40.30 μH, 39.46 μH, 38.90 μH, 38.54 μH and 38.33 μH respectively. The mutual inductance, M are 1.323 μH, 1.214 μH, 1.159 μH, 1.126 μH, 1.104 μH, 1.085 μH, 1.076 μH and 1.071 μH respectively. In square coil, the self-inductance is at 67.51 μH with M of 1.888 μH whilst the saggy square coil resulting in 84.68 μH with M of 2.366 μH.

6 Conclusion

This paper investigated the impact of saggy winding on the self-inductance and mutual inductance in inductive coupling power transmission. The outcomes from the simulation and experimental validations show the changes in self-inductance when the saggy winding with the catenary concept is introduced in the system. This is not only limited to the inductive power transmission system but could potentially be implemented for a much larger application with a similar concept. By taking into consideration the saggy coil factor, the performance of the inductive coupling system can be optimised. This could be used to predict the reliability and stability of the system. Future work will include the experimental setup of saggy coils with the driven board to observe the power transfer efficiency with the different geometrical shapes of the saggy coils.

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